

Investment Portfolio Opt Model

Investment Assets

Activo	Return	Liquidity	Risk	Term
Efectivo	None	Standard	High	None
Cuentas alto rendimiento	Fixed	High	Low	Short
CDT	Fixed	Mid - Low	Low	Short - Mid
Stocks - Acciones	Variable	High	Mid - High	Mid - Long
Exchange-Traded Funds (ETFs)	Variable	High	Mid	Mid - Long
Indexes - Índices	Variable	High	Mid	Mid - Long
Real Estate Investment Trust (REIT)	Variable	High	Mid	Mid - Long
Gold - Oro	Variable	Mid	Low	Value Store
Cryptos	Variable	High	High	Value Store

An investor holds total investable capital W and wishes to allocate it across N pre-screened financial instruments:

ETFs

Broad market exposure
with low cost

Bonds

Fixed income, stability
and drawdown buffer

Stocks

Alpha generation
and growth potential

Spanning three geographic regions:

United States

Deepest markets

European Union

Developed diversifier

Emerging Markets

High growth potential

Why Linear? Markowitz is quadratic. A linear formulation uses risk proxies (beta caps, CVaR) keeping the model tractable, transparent, and extensible.

Compact LP Formulation

$$\max \quad Z = \sum_{i \in I} (r_i - f_i) \cdot x_i$$

subject to:

$$\sum x_i = W$$

[Budget]

$$x_i \leq U_i \cdot W$$

[Concentration]

$$L_c \cdot W \leq \sum x_i \leq U_c \cdot W$$

[Asset-class bands]

$$L_g \cdot W \leq \sum x_i \leq U_g \cdot W$$

[Geographic bands]

$$\sum \beta_i \cdot x_i \leq \beta^{\max} \cdot W$$

[Beta ceiling]

$$\sum \sigma_i \cdot x_i \leq \sigma^{\max} \cdot W$$

[Volatility ceiling]

$$\sum d_i \cdot x_i \leq D^{\max} \cdot W$$

[Liquidity floor]

$$\eta + 1/((1-\alpha) \cdot S) \cdot \sum s_s \leq \text{CVaR}^{\max} \cdot W$$

[Linearized CVaR]

$$x_i \geq 0 \quad \forall i \in I$$

[Non-negativity]

Index Sets & Input Params

Index Sets

Symbol	Description	Example
$i \in I$	All prescreened instruments ($N = I $)	SPY, AGG, VOO, BUND ...
$c \in C$	Asset classes	{ETF, Bond, Stock}
$g \in G$	Geographic regions	{US, EU, EM}
$I_c \subseteq I$	Instruments in asset class c	$I_{\text{Bond}} = \{\text{AGG, BND, ...}\}$

Input Parameters

Param	Unit	Description
W	USD	Total investable capital (budget)
r_i	decimal	Expected return of instrument i
β_i	unitless	Market beta (systematic risk proxy)
σ_i	decimal	Annualized volatility of instrument i
f_i	USD	Transaction cost per unit invested

Decision Variables

x_i = dollar amount allocated to instrument i , $\forall i \in I$

Equivalently: $w_i = x_i / W$ (portfolio weights). Dollar amounts preferred because constraints on lot sizes and fees are more natural in monetary units.

Auxiliary Variables

$$s_i \geq 0$$

Slack for instrument i 's contribution to linearized CVaR constraint

$$\eta$$

VaR threshold auxiliary in CVaR formulation (Rockafellar & Uryasev, 2000)

$$y_i \in \{0,1\}$$

Binary indicator for instrument inclusion (elevates to MILP if used)

Cost Function

Maximize the net expected return of the portfolio

$$\text{Maximize } Z = \sum_{i \in I} (r_i - f_i) \cdot x_i$$

r_i

Expected return rate from analyst forecasts, factor models, or historical means

f_i

Proportional transaction cost (commissions, bid-ask spread, fees)

x_i

Dollar allocation — the decision variable the solver optimizes



The quality of r_i estimates is the single most impactful input. Garbage in, garbage out.

Constraints

Two tiers of constraints govern the feasible region: necessary (physical validity) and policy-driven (risk appetite and mandates).

NECESSARY

C-1 Budget

$$\sum x_i = W$$

Every dollar deployed or held as cash.

NECESSARY

C-2 Non-Negativity

$$x_i \geq 0 \quad \forall i$$

No short-selling (unless authorized).

POLICY

C-3 Concentration

$$x_i \leq U_i \cdot W$$

Max 10% per single instrument.

POLICY

C-6 Beta Ceiling

$$\sum \beta_i \cdot x_i \leq \beta^{\max} \cdot W$$

Bounds systematic risk linearly.

Diversification Constraints

C-4 ASSET-CLASS BANDS

$$L_c \cdot W \leq \sum x_i \leq U_c \cdot W \quad \forall c \in C$$

Asset Class	Min (L)	Max (U)	Rationale
ETFs	20%	60%	Broad diversification core; low cost
Bonds	15%	50%	Income and drawdown buffer
Stocks	10%	50%	Alpha generation; higher return potential

C-5 GEOGRAPHIC BANDS

$$L_g \cdot W \leq \sum x_i \leq U_g \cdot W \quad \forall g \in G$$

Region	Min (L)	Max (U)	Rationale
United States	30%	70%	Deepest markets; benchmark anchor
European Union	10%	40%	Developed-market diversifier; EUR exposure
Emerging Markets	5%	25%	Higher growth potential; higher tail risk

Risk & Liquidity Constraints

C-7 Weighted Volatility Ceiling

$$\sum \sigma_i \cdot x_i \leq \sigma^{\max} \cdot W$$

Linear approximation of portfolio risk using individual volatilities. **Ignores correlations but provides a useful upper bound.**

C-8 Liquidity Floor

$$\sum d_i \cdot x_i \leq D^{\max} \cdot W$$

Ensures portfolio maintains minimum average liquidity to meet redemption or margin requirements.

C-9 Linearized CVaR (Advanced)

$$\eta + 1/((1-\alpha) \cdot S) \cdot \sum s_{-s} \leq \text{CVaR}^{\max} \cdot W$$

Rockafellar-Uryasev formulation: controls tail risk while keeping the problem fully linear via scenario auxiliary variables.

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